Design of a Fractional Order PD Controller Tuned by Firefly Algorithm for Stability Control of the Nonlinear Ball and Plate System

4 authors:

Manashita Borah
University of California, Berkeley

Lalbahadur Majhi
College of Engineering and Technology

Prasanta Roy
National Institute of Technology, Silchar

Binoy Krishna Roy
National Institute of Technology, Silchar

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Manashita Borah[1], Lalbahadur Majhi[2], P. Roy[3], B.K.Roy[4]
[1,2,3,4]Department of Electrical Engineering, National Institute of Technology, Silchar, India
[1]manashitaborah@gmail.com, [2]lbmajhi@gmail.com, [3]proylinux@gmail.com, [4]bkrnits@gmail.com

Abstract*— In this paper, the author has proposed a Fractional Order Proportional Derivative (FOPD) controller tuned by Firefly Algorithm, that has a high convergence rate, robustness and the capability to attain global optimum in less number of iterations. The facts and contributions distinguishing this work are: Firstly, the simulation has been carried out without linearizing the Ball and Plate system, thus maintaining the physical nonlinearity intact in the system. Secondly, the IOPD (Integer Order Proportional Derivative) controller designed has been tuned by Firefly Algorithm, and provides improved results compared to the existing controller inbuilt in the system. Thirdly, the model (33-240, Feedback Instruments) taken for simulation is a newer version of Ball and Plate System, with not much research work done on it till date. Hence, comparisons of the results obtained have been done with that of an older model (CE151, Humusoft).The comparison has also been carried out with the PD controller already inbuilt in the system. And lastly an FOPD is designed using the *fod* library of FOMCON toolbox and results are compared with all of the above.

***Keywords— Ball and Plate System, Fractional Order Proportional Derivative (FOPD) controller, Firefly Algorithm (FA), FOMCON*.**

I. INTRODUCTION

The Ball and Plate (BP) system allows the motion of a hollow ball along a particular trajectory on a lightweight plate, the tilting of which is adjusted in two perpendicular directions. It has for long attracted the interest of researchers especially because of its inherent nonlinearity and instability, both of which makes control and identification a challenging task.

Stabilization control of the BP system comprises of holding the ball at a specified position on the plate. Trajectory tracking control requires that the ball follow the given path of the desired trajectory [1].Trajectory tracking control on this system has been achieved by hierarchical fuzzy control, simple fuzzy and mechatronic principles in[2,3,4], FOPD tuned by Genetic Algorithm [5], Naslin's PID [6], touch screen, FCMAC and machine vision respectively in [7,8,9].The drawbacks with the various previous control schemes are that, either they linearised the multivariable non linear system or have neglected the double feedback loop structure, while, switching mechanism adopted [10] is inconvinient, else the optimisation algorithm used provided a slow convergence or got centred around local optimum value, besides accurate identification as in [11] is a herculean task. This paper aims to inculcate all of the above problems, and design an effective controller that can tackle them.

The organisation of this paper is as follows: the present Section provides a brief introduction and literature review on the Ball and Plate System, Section II gives its nonlinear modelling, Section III gives the basics of FA and its methodology to design the IOPD, Section IV gives a brief description of the tuned FOPD and finally Section V throws light on the results and comparisons drawn with various existing PD controllers, followed by a conclusive summary comprising of future scope of work in Section VI.

II. BALL AND PLATE SYSTEM

A brief system investigation and mathematical modelling of the plant is given below:

A. System Model

The model considered 33-240, is even more interesting due to the presence of an electromagnetic actuation table [2] unlike stepper motor, servo motor, linear solenoid actuators present in the older models.

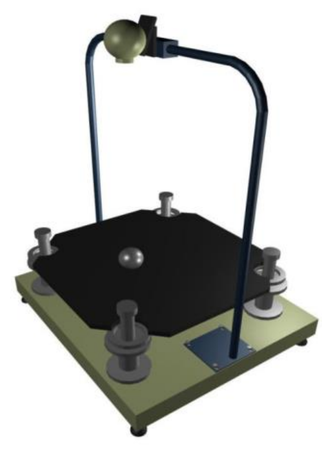

Fig. 1. Ball and Plate system, model no.33-240

B. Mathematical modelling

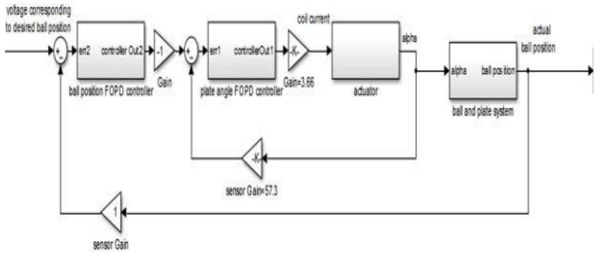

Fig. 2. Ball and Plate block diagram

The nonlinear model of the Ball and Plate system constitutes of the following equations as described as in [12].

$$\tau_1 = 0.2K_c \frac{(i_1+2.2)^2}{|\alpha-0.17|} - 0.2K_c \frac{(i_1-2.2)^2}{|\alpha+0.17|}, |\alpha| < 0.17 \quad (1)$$

$$mg\sin\alpha + x(m + \frac{I_b}{r^2}) - m(x\alpha + y\beta)\alpha = 0 \quad (2)$$

$$\tau_2 = 0.2K_c \frac{(i_1+2.2)^2}{|\beta-0.17|} - 0.2K_c \frac{(i_1-2.2)^2}{|\beta+0.17|}, |\beta| < 0.17 \quad (3)$$

$$mg\sin\beta + y(m + \frac{I_b}{r^2}) - m(x\alpha + y\beta)\beta = 0 \quad (4)$$

Where, I_b-moment of inertia (kg/m^2), g- gravitational constant (m/s^2) K_c-constant (Nm-deg/A^2), x- ball position in X direction(m), - plate angle from X axis(rad), $_1$-torque for X axis(Nm), i_1- inductor's current for generating $_1$; y, , $_2$, i_2 are the corresponding values w.r.t Y axis. Compared to the plate dynamics, the ball mass is very small and thus acceleration generated by the ball is negligible, so the MIMO model is decoupled and represented as two SISO models, with X and Y position controlled independent of each other. The simulink block diagram of the system for controlling the X position of the ball has been shown in Fig.2 using (1) and (2). The block diagram for controlling the Y position of the ball can be constructed similarly from (3) and (4).

III. FIREFLY ALGORITHM (FA) FOR IOPD (INTEGER ORDER PROPORTIONAL DERIVATIVE) CONTROLLER

When compared to ZN method, FA results in a superior system performance in terms of time domain specifications as proved in [13]. The flashing characteristics of real butterflies are the following as in[14]:

1) Owing to their unisexuality, the fireflies move towards the more attractive and brighter ones regardless of their sex**.**

2) The degree of attractiveness of a firefly is proportional to its brightness which decreases as the distances from the other fireflies increase.

3) If there is not brighter or more attractive firefly then a particular one, then it will move randomly.

TABLE I. MAIN FA PARAMETERS

SL no.	Designation	parameters	values
1	no. of fireflies	nf	5
2	no. of generations	ng	100
3	randomness	α	0.5
4	initial attractiveness	min	0.2
5	absorption coefficient	γ	0.1
6	no. iterations	N= nf* ng	500

Owing to considerable overshoots in an inherently unstable system, the ISE values are excessively enlarged. Henceforth the fitness function is chosen as the performance index ISE, which is to be minimised.

Fitness Function= Min. $e^2(t)dt$.

Following the flowchart shown Fig.3 , the tuned results obtained for designing the IOPD controller with their corresponding ST(settling time) and OS (overshoot) shown in row 5 of Table II

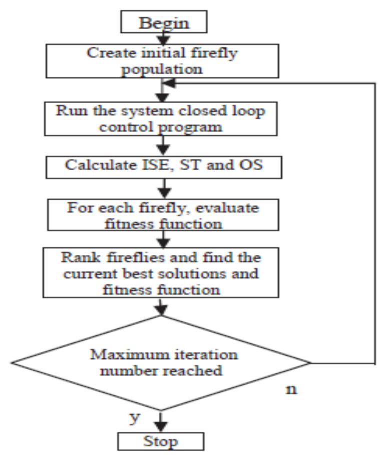

Fig. 3. FA Flowchart

IV. FOPD CONTROLLER

An FOPD controller [15] in time domain can be represented as :

$u(t)= K_p\, e(t) + K_d D^{\mu} e(t)$, (μ>0),

where $u(t)$ is control signal and $e(t)$ is the error signal, and its transfer function is $G_c(s)= K_P+K_D s^{\mu}$.

Due to the additional tunable parameter μ (fractional order of the derivative controller) of the FOPD, the real world systems can be controlled with further accuracy compared to an IOPD. The most popular amongst all approximation methods of fractional differential equations has been proposed by Oustaloup, which is used in this paper. Due to the commutative property of the fractional operator s^a, a fractional order a 1 can be approximated by $s^a = s^n s^q$, where $n = a - q$

denotes the integer part of a and s^q is obtained by the Oustaloup approximation in a specified frequency range ($_l$, $_h$) and of order N by a rational transfer function obtained in [16]. The order of approximation is taken as 5 valid in the frequency range (0.001,1000) rad/sec. It is found that for certain values of µ, the controller exhibits increased oscillations. Thus, the author has fine tuned the derivative constants and μ using the *fod* library of FOMCON Toolbox [17]. Due to presence of the measurement noise, produced by the D-controller, in controlling the ball position, an L.P.F is introduced with filter coefficient N_f = 176. The results are as shown in row 6 of Table II.

V. RESULTS AND DISCUSSIONS

The prime contribution of this paper has been the significant reduction in settling time in the Nonlinear model of the system, as distinctly shown in rows 5 and 6 of table II. The results of the step responses in terms of overshoot (OS) and settling time (ST) have been compared with those PD's designed after linearising the Ball and Plate system. The desired specifications have been OS< 20 % and ST< 4.5sec. It is worth mentioning that the simulations for [18] have been carried out considering the corresponding transfer function put forward in [19]. And even though the Naslin PD controller provides the least overshoot, yet it fails to meet the desired specification of settling time. Besides, the PD controller designed by GL method gives more undershoot than overshoot. The FOPD controller not only meets the desired specification but also provides improved results conmpared to its corresponding IOPD controller. Fig. 4 shows the comparisons of the FA tuned PD and FOPD with the PD's designed for linearised model as in [18], Fig. 5 shows the circular trajectory tracked by system PD and the proposed PD controllers. Fig. 7 is a magnified view from (0-10) seconds of Fig. 6, which shows the comparison of step responses of the system PD with that of the proposed ones. Fig. 9 is a magnified view from (0-1) seconds of Fig. 8 where the sine waves are being tracked. In all of the above the FOPD controller performs better than those with whom it was compared.

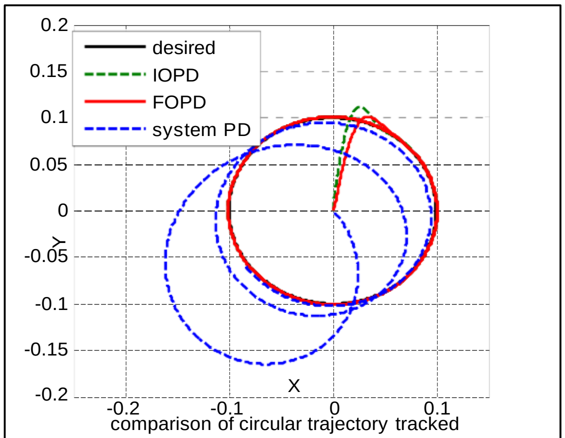

Fig. 5 Circular trajectory tracked by the system PD, IOPD, FOPD controllers

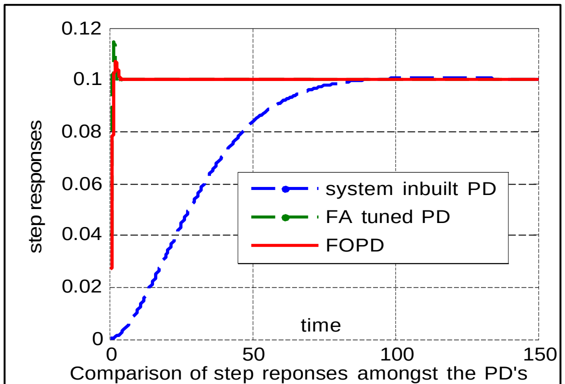

Fig. 6 Step response comparison with the system PD, IOPD, FOPD controllers

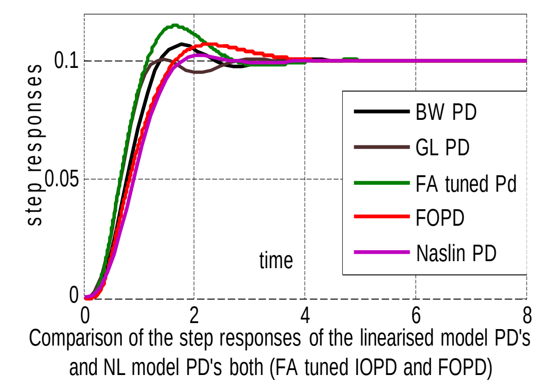

Fig. 4 Step response comparison

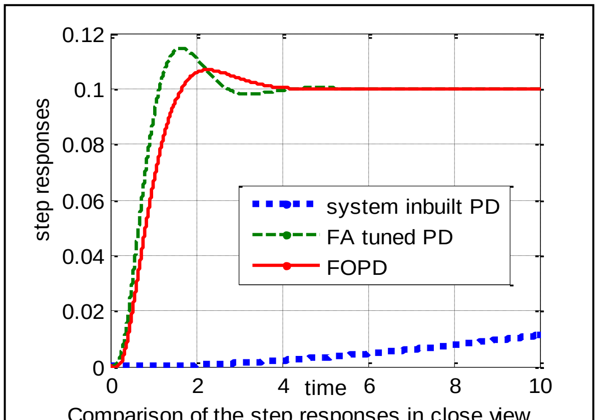

Fig. 7 Step response comparison with the system PD, IOPD, FOPD controllers in close view

TABLE II. COMPARISON OF VARIOUS PD CONTROLLERS FOR LINEARISED MODEL WITH THE PROPOSED ONES

SL No	**Ball And Plate**	**Model no:**	**PD design method**	**PD type**	**K_p**	**K_d**	**µ**	**N_f**	**OS (%)**	**ST (sec)**
1	Linearised	CE 151	Butterworth (BW) [18]	IOPD	0.7436	0.5562	1	0	7	5
2	Linearised	CE 151	Graham Lanthorp (GL) [18]	IOPD	1.110	0.781	1	0	-9.5	4.8
3	Linearised	CE 151	Naslin [18]	IOPD	0.5587	0.5057	1	0	3	5.5
4	Nonlinear	33-240	Inbuilt PD in the system [12]: Outer loop controller Inner loop controller	IOPD	44.00; 0.3	959.904 2.772	1	0	0	120
5	Nonlinear	33-240	Firefly Algorithm (FA) Optimization: Outer loop controller Inner loop controller	IOPD	40.7334; 18.001	0.4985; 0.0609	1	0	15	4.4
6	Nonlinear	33-240	Fine-tuned by FA, FOMCON Outer loop controller Inner loop controller	FOPD	40.7334; 23.98	0.4985; 0.0909	1.089; 1.001	176 ;0	7	4.2

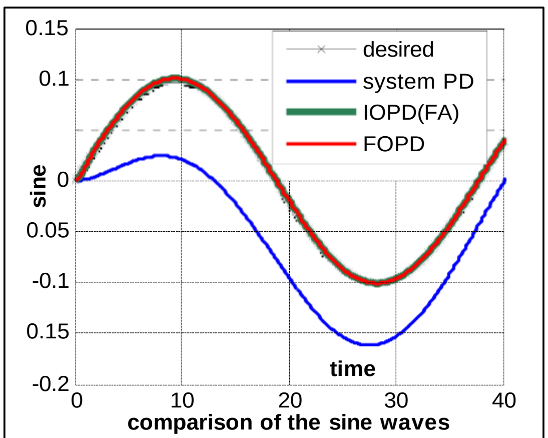

Fig. 8 Sine waves tracked by the system PD, IOPD, FOPD controllers

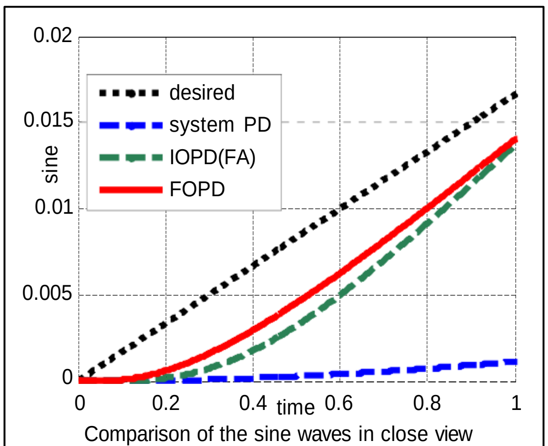

Fig. 9 Sine waves tracked in close view

VI. CONCLUSIONS

As distinctly depicted in the results shown, the proposed FOPD, reduces the settling time significantly as compared to the PD inbuilt in the Ball and Plate system, without linearisation. In addition to this, the results are comparable with those obtained by linearising the model and exhibit small overshoot and reduced settling time.This paper applied the FA and FOMCON toolbox to tune the FOPD parameters, and the control effect is found to be excellent.The limitaions are: the cross coupling between the two perpendicular directions of the plate has been neglected. The ball acceleration has been neglected, owing to its small mass. The motion between the ball and plate is assumed to be frictionless.

Future scope of work : The controllers can be checked for the ball tracking a desired trajectory apart from the circular one, and different range of trajectories can be explored and compared. The results can also be compared on the basis of errors which was minimised. All real world systems exhibit friction in motion. The idea of frictionless assumption is for simplification purposes. Hence there lies a scope of designing an effective controller considering the frictional force available in motion and also of overcoming the above mentioned limitations.

REFERENCES

[1] X. Dong, Z. Zhang and C. Chen, "Applying genetic algorithm to on-line updated PID neural network controllers for ball and plate system," 4th Int. Conf. on Innovative Computing, Information and Control, China, IEEE, 2009, pp. 751 -755.

[2] X. Fan, N. Zhang, and S. Teng, "Trajectory planning and tracking of ball and plate system using hierarchical fuzzy control scheme," Fuzzy Sets and Systems, Elsevier, vol. 144, Issue 2, 2004, pp. 297–312.

[3] Rastin, M.A. ;Talebzadeh, E. ; Alaeddin, M.,"Trajectory tracking and obstacle avoidance of a ball and plate system using fuzzy theory", 13th Iranian Conference on Fuzzy Systems (IFSC), IEEE, Qazvin, 2013, pp.1-5

[4] S. Awtar, C. Bernard, N. Boklund, A. Master, D. Ueda and Kevin Craig, "Mechatronics Design of a ball-on-plate balancing system," Mechatronics, Elsevier, vol. 12, Issue 2, 2002, pp. 217-228.

[5] Yu Zhong, A. Dutta, C. Copot, Clara-Mihaela Ionescu and R. De Keyse, "Implementation of a fractional PD controller tuned by genetic algorithm for a Steward platform", 9th Asian Control Conf. (ASCC), IEEE, 2013, pp. 1-6.

[6] A. Jadlovska, S. Jajcisin and R. Lonscak, "Modelling and PID control design of nonlinear educational model Ball & Plate," 17th International Conference on Process Control, 2009, pp. 475-483.

[7] De-hu Yuan,"Pneumatic servo ball & plate system based on touch screen and oscillating cylinder," Intelligent Systems and Applications, IEEE, 2009, pp. 1-4.

[8] Marco A. Moreno-Armendariz, Cesar A. Perez-Olvera, Floriberto Ortiz Rodrıguez and Elsa Rubio, "Indirect hierarchical FCMAC Control for the ball and plate system," Neurocomputing, Elsevier, vol. 73, Issues 13–15, 2010, pp. 2454 -2463

[9] Li Yuan ,W. Qinglin ; L. Qinwu ; D. Nanhua, "On machine vision based ball and plate Labyrinth system", 31st Chinese Control Conference (CCC), IEEE, Hefei, 2012, pp. 5152 – 5156

[10] F. Zheng, X. Li ; S. Wang, D. Ding, "Position control of ball and plate system based on switching mechanism", IEEE Int. Conf. on Automation and Logistics (ICAL), Chongqing, 2011,pp. 233 – 237

[11] Y. Wang, X. Li ; Y. Li ; B. Zhao, "Identification of ball and plate system using multiple neural network models", Int. Conf. on System Science and Engineering (ICSSE), IEEE, Dalian, 2012, pp. 229 - 233

[12] Ball and Plate Control Experiments 33-240-LabView, 1st ed., Feedback Instruments Ltd, East Sussex, UK, 2011, pp. 2- 14

[13] O. Bendjeghaba, S. IshakBoushaki and N. Zemmour, "Firefly Algorithm for optimal tuning of PID controller parameters," 4th Int. Conf. on Power Engineering, Energy and Electrical Drives, IEEE, 2013, pp. 1293-1296.

[14] X. S. Yang, "Firefly Algorithm", in Nature-inspired Metaheuristic Algorithm, 2nd ed., UK, Luniver Press, 2010, ch. 10, pp. 81-89.

[15] M. Reza Faieghi and Abbas Nemati , On Fractional-Order PID Design, ch.13, 2011.

[16] A. Oustaloup, F. Levron, B. Mathieu, and F. M. Nanot, "Frequency-Band Complex Noninteger Differentiator: Characterization and Synthesis," IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 47, no. 1, , 2000, pp.25-39.

[17] A. Tepljakov, E. Petlenkov and J. Belikov, "A Flexible MATLAB Tool for Optimal Fractional-order PID Controller Design Subject to Specifications," 31st Chinese Control Conference (CCC), IEEE, 2012, pp. 4698-4703.

[18] A. Jadlovska, S. Jajcisin and R. Lonscak, "Modelling and PID control design of nonlinear educational model Ball & Plate," 17th International Conference on Process Control, 2009, pp. 475-483.

[19] CE 151 Ball and Plate Apparatus, Educational Manual , Humusoft , 1996-2008.